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An Irrationally Rational Game Model

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ABSTRACT

In the application of game theory into the scenario of selling and buying a product in real life, sellers and buyers concern not only the acquisition utility of the good but also the transaction utility. To this end, in this paper we develop a game model with payoff matrix of aggregating transaction utility and acquisition utility, where we set the perceived transaction utility of a customer according to prospect theory. Moreover, we also study how the equilibrium of a game of this kind is influenced by some irrational factors that can be reflected by transaction utility. Finally, we use our model explain why online promotion selling in Tmall.com on Singles' Day is so successful in China.

Keywords: Game theory, transaction utility, prospect theory, price discount, electronic commerce.

INTRODUCTION

Game theory [17] is an effective tool to help the players in a game rationally choose strategies that can maximize their individual utilities. In particular, in a trading game a buyer pays for a good just because buying can bring more profits to the buyer than not buying. However, situations often differ in real life. Impulsive buying often happens to almost everyone when they are offered tremendous discount in a sale promotion, which can be loosely defined as "special offer", essentially aiming to stimulate demands during a certain period of time [16]. For example, a lady might not be able to resist the temptation of buying a lovely gown when it is 50% off, even though she would regret that it costs her one month's salary. It is irrational if we still think the utility of a good is simply the acquisition utility of the good, which depends only on the value of the gown received compared with the price. Actually, this kind of impulsive buying takes up the majority purchases in shopping mall and supermarkets, and in some situation, it even covers 80% of sales [27]. Then why does it often happen?

To answer the question, this paper will develop a new game model that can put such an irrational factor into account. More specifically, we will build a game model based on transaction utility theory [23] and prospect theory [8]. The transaction utility theory reveals that the utility of a good should be an aggregation of its acquisition and transaction utilities. The former one depends only on the value of the good received compared to the outlay, which is similar to the concept of consumer surplus; while the latter one depends solely on the perceived merits of the deal. Ann and Yuri [1] evidence the existence of transaction utility by an on-line auction experiment, which shows that people bid more for an item with a higher posted "buy now" price than for an identical item with a lower posted "buy later" price. Another experiment of psychology [27] also shows that the bigger the transaction utility is, the more purchases the subjects tend to make. Therefore, in our model the payoff that a player takes a strategy is the aggregation of the perceived transaction utility and acquisition utility. Moreover, for the individual customers during a transaction, according to prospect theory [8] they perceived utility in a special non-linear way. Accordingly, we set the buyer's transaction utility in the fashion of the value function model of prospect theory [8]. In addition, using our model we discuss how some rational and irrational factors influence the equilibrium of a game of this kind, so that the marketing manager of an online company can better understand that for each kind of customers, how much discount is enough for stimulating them to buy.

With our new irrational rational game model, we can better understand why online shopping is so popular. That is, people see a price of a good when they are on high streets; but when they browse the Internet, they often find the same product but in a lower price. Thus, the price difference turns into a perceived transaction utility, which makes them buy the good impulsively. The success of "Tmall.com Singles' Day Festival" is a very convincing piece of evidence for the effect of transaction utility (we will discuss in details in the illustration section). Another evidence is that one of the main reasons why university students in China shopping online is low-price [28].

The rest of this paper is organized as follows. Firstly, we recap and discuss relevant concepts and notations in transaction theory and prospect theory. Secondly, we define our game model. Thirdly, we reveal some properties of the model. Fourthly, we employ the model to explain the success of Tmall.com on Singles' Day Festival in China. Fifthly, we discuss some related work to show clearly how our work advances the state-of-art. Finally, we conclude the paper with some future research directions.

PRELIMINARY

This section will recap and discuss the relevant concepts and notations in transaction theory [23] and prospect theory [8], which we need to build up our game model.

Buyer Side

We first discuss different situations for individual customers. The transaction utility of a good mainly comes from the customer's personal experience of buying it and its reference price perceived from the environment [12], including the original price label, the salesman's verbal cues, the price presented in the official website, and so on. The acquisition utility of a good is determined mainly by the reservation price and the selling price of the good. Behavior experiments [9] suggests that subjects who played the role of buyers in a price negotiation adopted their reservation price as a reference point. And during the purchasing period, it would be perceived as gains or losses by the buyer depending on whether the selling price is lower or higher than the reference point.

To allow different weights, in [23] a buyer's total utility function of a good z is defined as follows:

$$w_b(z, p_b, p_b^*) = v(\overline{p}_b, -p_b) + \beta_b v(-p_b : -p_b^*). \quad (1)$$

where p_b is the price the buyer paid for good z; p_b^* is the reference price for z, which is an expected or "just" price for z; $\overline{p}_b$ is the value equivalent to z, the amount of money which would leave the individual indifferent between $\overline{p}_b$or z as a gift; β_b represents how much the buyer weights the transaction utility; $v(\overline{p}_b, -p_b)$ represents the buyer's acquisition utility; and $v(-p_b : -p_b^*)$ represents the buyer's transaction utility. Actually,$\beta_b$is the degree to which a buyer cares about money (see the fourth section for detailed discussion); and $\overline{p}$ can be understood as the reservation price in microeconomics [5] (i.e., it is the highest price that a buyer is willing to pay).

The transaction utility is gained by a customer only if the good is on sale, and the acquisition utility is gained by the customer only if the deal is made. So, we still need to differentiate the following situations: 1) For an individual customer, if one good is on sale and he bought it, then the customer gets both its transaction utility and acquisition utility. 2) If one good is not on sale and he does not buy it, then he gets neither of these two utilities. 3) If one good is not on sale and the customer bought it, he gets its acquisition utility but no transaction utility. 4) If one good is on sale but the customer has not bought it, then the customer gets no acquisition utility as he does not gain the product and cannot enjoy it. However, he did get a negative transaction utility because the good is on sale but he missed it. It is something like that at the moment when he made the decision, he missed a chance to save some money. It could cause pain in the customer's heart because he might regret even after a long time. Therefore, in this case, the transaction utility is negative. The four situations above can be summarized in Table 1, which is helpful when we calculate the utility of an individual buyer.

Table 1: The Utilities of Buyer

Situation	Utilities
Sale & Buy	Acquisition utility + Transaction utility
Sale & Not buy	-Transaction utility
No sale & Buy	Acquisition utility
No sale & Not Buy	0

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No sale & Not Buy	0

Further, prospect theory reveals that the utility perceived is not a linear, but non-linear, more specifically, S-shaped curve, in which the marginal gain decrease and the pain of losing an amount of money is larger than the happiness of gaining the same amount of money. So for individual buyers, a more realistic representation of utility should be figured out. In [24], Tversky and Kahneman advocated a form of value function. Accordingly, we define the following perceived transaction utility function of a customer with the value function.

Definition 1.
The perceived transaction utility t_b^p of buyer is the transaction utility abuyer perceived. It is defined as follows:

$$t_b^p = \begin{cases} (t_b)^{\alpha} & \text{if } t_b \geq 0 \\ -\lambda(-t_b)^{\alpha^t} & \text{if } t_b < 0 \end{cases} \quad (2)$$

where $t_b = \beta_b v(-p_b : -p_b^*)$ is the actual transaction utility when a consumer purchases good in a sale, $\alpha \in (0, 1)$ and $\alpha' \in (0, 1)$ are risk attitude coefficient, and $\lambda \in (1, \infty)$ is loss aversion coefficient.

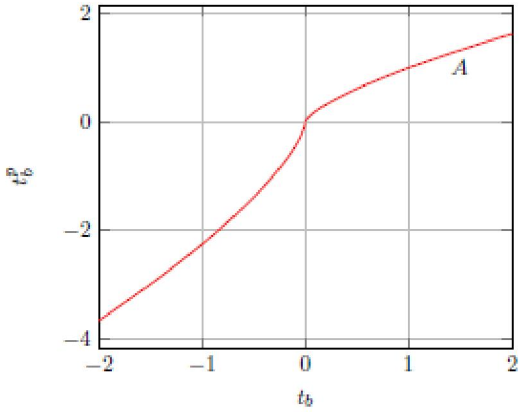

Figure 1: The relationship between t_b^p and t_b

The reference point of the perceived transaction utility function is the p_b^*. We can make a simple example to see the relationship between t_b^p and t_b. In Fig. 1, line A represents t_b^p, whose $\alpha = \alpha' = 0.7$, $\lambda = 2.25$. From the Fig. 1, we clearly see that for the customer, the increased speed of perceived transaction utility become smaller when the total amount of transaction utility increased. Meanwhile, gaining the same amount of transaction utility causes less perceived transaction utility than losing the same amount of transaction utility. Thus we defined the buyer's utility function with acquisition utility and perceived transaction utility as:

$$u_b(z, p_b, p_b^*) = v(\overline{p}_b, -p_b) + t_b^p. \tag{3}$$

Seller Side

Now we turn to discuss the seller side. Inspired by Thales's total utility function for the customers we mentioned above, we define the total utility of a seller as follows:

$$u_s(z, p_s, p_s^*) = v\left(-\underline{p}_s, p_s\right) + \beta_s v(-p_s^*: -p_s), \tag{4}$$

wherep_s is the price the buyer paid for good z; p_s^* is the seller's reference price of the good, for example, its price before discount; $\underline{p}_s$ is the cost of good z for the seller; β_s indicates how much the seller weights the transaction utility; $v\left(-\underline{p}_s, p_s\right)$ is the acquisition utility; and $v(-p_s^*: -p_s)$ is the seller's transaction utility.

Sometimes it is reasonable to put the transaction utility into account of a seller. For example, Raghubir and Corfman's experiments [20] showed that under some specific conditions price promotions affect pretrial brand evaluation and do so unfavorably. They also found that consumers tend to perceive negatively when promotions are uncommon in the industry. Moreover, frequent or large price promotion will damage brand equity by bringing bad influence to consumer's loyalty and perceived value [15, 18]. On the contrary, when a company prices a good increasingly, it often delivers the message that its products are of high quality and thus makes the company's reputation higher. So, by means of formula (4), we define that transaction utility is negative when lowering the price, while the transaction utility is positive when increasing. However, for the sake of space, in this paper, we focus on the situation of price decreasing situation.

Specifically, we have: 1) If their good is on sale and a customer buys it, the company gets both transaction utility and acquisition utility. 2) If their good is not on sale and no customers buy it, their acquisition utility value is negative for the company has invested money to produce the good but gets no return. However, as they have not yet offered discount, so the value of transaction utility is zero. 3) If their good is not on sale and a customer bought it, they get the same acquisition utility but no transaction utility. 4) If their good is on sale but no customers have bought it, they get both acquisition utility and transaction utility. It gets acquisition utility because producing the good causes them some cost but gets no return, and they gain a transaction utility because they indeed offered discount, which influences its reputation. The above four situations are summarized in Table2.

GAME MODEL DEFINITION

This section will define our game model of discount trading.

Definition 2.

A discount trading game is a 4-tuple of (N, S, U, M), where:

-N ={1,2},where1representsaseller and2representsabuyer.

-$S = S_1 \times S_2$*, where* $S_i = \{s_{i,1}, s_{i,2}\}$ *is the strategy setoff player* $i \in N$*. And* $s_{1,1}$ *is "Sale" and* $s_{1,2}$ *is "No Sale";* $s_{2,1}$ *is "Buy" and* $s_{2,2}$ *is "Not Buy".*

-U =$\{u_i(s_{1,j}, s_{2,j'}) | i \in N\}$*, where* $u_i(s_{1,j}, s_{2,j'})$ *represents the utility of player i when player 1 chooses* $s_{1,j} \in S_1$ *and then the player 2 chooses* $s_{2,j'} \in S_2$*, and* $u_1(s_{1,j}, s_{2,j'})$ *is given by formula (4), and* $u_2(s_{1,j}, s_{2,j'})$ *is given by formula (3).*

-M= $(s_{1,j}, s_{2,j'})$ *indicates this game proceeds as follows: firstly seller 1 takes strategy* $s_{1,j} \in S_1$ *and then buyer 2 takes strategy* $s_{2,j'} \in S_2$*.*

We will use $(s_{1,m}, (s_{2,m_1'}, s_{2,m_2'}))$ to represent that player 2's strategy is decided by player 1's choice $s_{1,m}$. That is, when player 1 chooses $s_{1,1}$, player 2 will choose $s_{2,m_1'}$, and when player 1 chooses $s_{1,2}$, player 2 will choose $s_{2,m_2'}$, as shown in Fig. 2. From the game tree, we can see clearly how player 2's moves connects to player 1's.

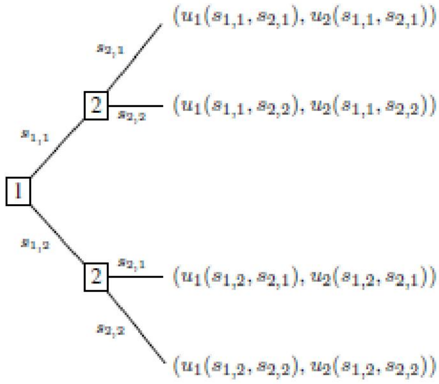

Figure 2: The game tree of our model

From Fig. 2, we can see that the game defined above is a perfect information dynamic game [26]. The seller first decides to offer a discount or not, and then the customer decides to buy or not. Before making a decision, the customers clearly know what discount they were offered and how much money they should pay. Each player in the game will choose the optimal strategy according to the utility that they can gain. Formally, we have:

Definition 3.

An acquisition-transaction equilibrium ς to discount trading game G is (s_1^*, s_2^*), where $s_1^* \in S_1$, $s_2^* \in S_2$ satisfying:

$$\forall s_1' \in S_1, s_1' \neq s_1^*, u_1(s_1^*, s_2^*) \geq u_1(s_1', s_2') \tag{5}$$

$$\forall s_2'' \in S_2, u_2(s_1', s_2') \geq u_2(s_1', s_2''),\ u_2(s_1^*, s_2^*) \geq u_1(s_1^*, s_2'') \tag{6}$$

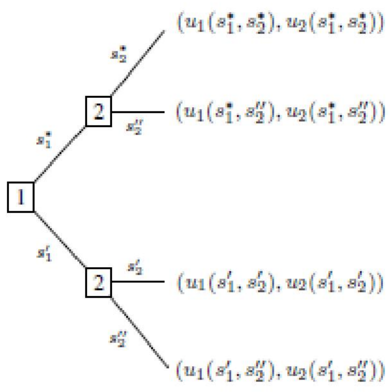

Figure 3: Backward induction for finding the Nash equilibrium

From Fig. 3, we can understand Definition 3 better. In every sub game of the discount trading game G, for the buyer, $s_2'$is the better choice. Then the seller can choose his better choice s_1^* according to the pay-off. In the sub game where the s_1^* leads to, the dominant choice $s_1'$of the buyer is s_1^*.

Using our game model with utility functions (3) and (4), we can explain some phenomena well in real life. If acquisition utility is concerned, it just can reflect material payoff. This makes the game model not be able to explain the situation of impulsive buying something due to discount but still not worth that price, such as some luxury goods because it is known to almost everyone that a luxury good's material pay-off is not that much but owning a luxury good can symbolize a buyer's status and taste. For example, Chinese people pay tremendous money aboard to buy luxury goods with discount they cannot get in domestic markets, because the discounts stimulate them to buy those luxury goods that can symbolize their richness. It is the transaction utility, instead of the acquisition utility, that triggers them to do crazy shopping overseas.

Theorem 1.
For the buyer, let $v(-p_b{:}\,-p_b^*) = p_b^* - p_b$, and let $v(\overline{p}_b, -p_b) = \overline{p}_b - p_b$. Then the buyer's utility $u_b$of strategy profile $(s_{1,i}, s_{2,i},)$ can be positive even its corresponding acquisition utility is negative.

Proof. By formula (3), we have $u_b = v(\overline{p}_b, -p_b) + t_b^p$, and according to the premise, we have $v(\overline{p}_b, -p_b) < 0$. Now we should prove $t_b^p > -v(\overline{p}_b, -p_b)$ is possible. Since this paper focuses on price decreasing situation, we have $p_b^* - p_b > 0$. If $\beta_b > 0$, then$t_b = \beta_b(p_b^* - p_b) > 0$, so we have $t_b^p = (\beta_b(p_b^* - p_b))^\alpha$. We have:

$$v(\overline{p}_b, -p_b) + t_b^p > 0 \Leftrightarrow (\beta_b\,(p_b^* - p_b))^\alpha > -(\overline{p}_b - p_b)$$
$$\Leftrightarrow \beta_b > \frac{(\overline{p}_b - p_b)^{\frac{1}{\alpha}}}{p_b^* - p_b}.$$

It means when$\beta_b > \frac{(\overline{p}_b - p_b)^{\frac{1}{\alpha}}}{p_b^* - p_b}$, the gain of the perceived transaction utility is able to compensate the loss of the acquisition utility. But when$\beta_b < 0$, $\beta_b\,(p_b^* - p_b) < 0$, so $t_b^p = -\lambda(\beta_b\,(p_b^* - p_b))^{\alpha'} < 0$. And we have $t_b^p + v\,(\overline{p}_b - p_b) < 0$. In this case, the gain of the perceived transaction utility is unable to compensate the loss of the acquisition utility.

The following is straightforward by Definition 3.

Theorem 2.
For any game with transaction utility and acquisition utility, there exists an acquisition-transaction equilibrium.

PROPERTIES

This session will discuss how the outcome of our game is influenced by various factors. The result can tell a marketing manager that for what kind of customers, how much discount they should make to attract them to buy.

Now we discuss the different situation of β. For all the players in a game, the values of parameter β in their utility functions can fall into the following 3 categories: 1) $0 \leq \beta \leq 1$; 2) $\beta > 1$; and 3) $\beta < 0$. We first discuss the customers. For the customers in category 1), they care about not only money they may pay but also the value of the good itself equally or even more. This kind of customers tend to estimate the value of the good according to both its acquisition utility and its transaction utility. For the customers in category 2), they care about money so much that we can call them bargain hunters. That is, whatever they buy, they tend to choose the one that they can take advantage of. For the customers in category 3), they gain negative (instead of positive) transaction utility when the price is lower. Maybe these are vain customers who want to buy expensive good to manifest their taste or show off themselves to others, or they just simply take price changing as an important message of good's quality and hate lowing the quality, even though they can save some money.

The three categories of β can also be applied to the seller, depending on who is the one suffering from price changing. For the sellers in category 1), they may sell general consumers goods, and if lowering the price, their reputations will be damaged, but the acquisition utility is more or equally important. For the sellers in category 2), they produce goods, which values depend desperately on the company's reputation instead of the goods' functions. We can take luxury goods as an example. If a company tries to trigger the massive shopping by a huge price discount, their luxurious image will be ruined and none will believe that their product is highly valuable any longer. Category 3) means declining price will increase the company's reputation. This sort of companies might be completely new in a market and so try to use market penetration pricing strategy (i.e., setting low level of price to quickly build market share [25]). Thus, with the increasingly growing market share, they have a chance to make themselves known to more new customers and manifest a strong brand image.

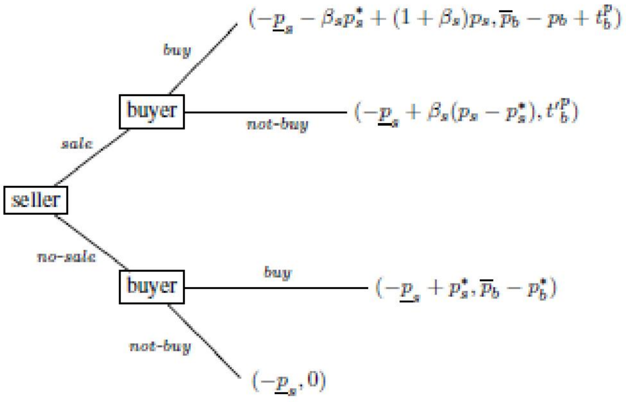

Figure 4: The game tree for property analysis

Now we examine the above discussion more accurately according to the game tree as shown in Fig. 4. Notice that the perceived transaction utilities of the buyer when buying or not buying are denoted as t_b^p and t'^p_b respectively, where formula (2) should be applied. Since we focus on price decreased situation and$t_b = \beta_b(p_b^* - p_b)$, $t'_b = -\beta_b(p_b^* - p_b)$. We have $t_b^p = (\beta_b(p_b^* - p_b))^\alpha$ if $\beta_b \geq 0$, $t_b^p = -\lambda(-\beta_b(p_b^* - p_b))^{\alpha'}$ if $\beta_b < 0$; and $t'^p_b = -\lambda(\beta_b(p_b^* - p_b))^{\alpha'}$if $\beta_b \geq 0$ and t'^p_b= $(-\beta_b(p_b^* - p_b))^{\alpha'}$ if $\beta_b < 0$.

Theorem 3.
Under all the three conditions, the payoff of the seller is always bigger when a customer chooses Buy than Not Buy.

Proof.
Firstly we compare the payoff of the seller between $(s_{1,1}, s_{2,1})$ and $(s_{1,1}s_{2,2})$, and then we compare the payoff of the seller between $(s_{1,1}, s_{2,1})$ and $(s_{1,1}s_{2,2})$.

If the seller chooses $s_{1,1}$, the seller's utility difference when the buyer chooses "Buy" or "Not-buy" is:

$$u_1(s_{1,1}, s_{2,1}) - u_1(s_{1,1}s_{2,2}) = \left(-\underline{p}_s - \beta_s p_s^* + (1+\beta_s)p_s\right) - \left(\underline{p}_s + \beta_s(p_s - p_s^*)\right) = p_s.$$

Since $p_s > 0$, when the seller chooses the strategy of sale, the seller's payoff will be bigger if the buyer chooses to buy.

If the seller chooses $s_{1,2}$, the seller's utility difference when the buyer chooses "Buy" or "Not-buy" is:

$$u_1(s_{1,2}, s_{2,1}) - u_1(s_{1,2}s_{2,2}) = \left(-\underline{p}_s + p_s^*\right) - (-\underline{p}_s) = p_1^*.$$

Since $p_s^* > 0$, when the seller chooses not to sale, the payoff will be bigger if the buyer chooses to buy.

Theorem 4. When a customer decides to buy,
if$0 \leq \beta_s \leq 1$or$\beta_s > 1$or$-1 < \beta_s < 0$, then it is better for the seller to choose no sale; and if $\beta_s < -1$, then it is better for the seller to choose sale.

Proof.
We can see the difference between $u_1(s_{1,1}, s_{2,1})$and $u_1(s_{1,2}s_{2,1})$ as follows:

$$u_1(s_{1,1}, s_{2,1}) - u_1(s_{1,2}s_{2,1}) = \left(-\underline{p}_s - \beta_s p_s^* + (1+\beta_s)p_s\right) - \left(-\underline{p}_s + p_s^*\right)$$
$= (1+\beta_s)(p_s - p_s^*)$.

Since we focus on the situation of price decreasing, we have $p_s - p_s^* < 0$. So, when $\beta_s > -1$, $(1+\beta_s)(p_s - p_s^*) < 0$. Thus,

$u_1(s_{1,1}, s_{2,1}) - u_1(s_{1,2}s_{2,1})< 0$. Then it is better for the seller not to choose sale. When $\beta_s< -1$, $(1 + \beta_s)\,(p_s - p_s^*)> 0$. Thus, $u_1(s_{1,1}, s_{2,1}) - u_1(s_{1,2}s_{2,1})> 0$. Then it is better for the seller to choose sale.

Theorem 5.

Table 3: The Equilibriums under Different Conditions

IF			THEN
$\overline{p}_b$ and p_b^*	β_b guarantees	β_s satisfies	Equilibrium
$\overline{p}_b > p_b^*$	$\overline{p}_b - p_b + t_b^p > t'^p_b$	$\beta_s > -1$	(No Sale, Buy)
		$\beta_s < -1$	(Sale, Buy)
	$\overline{p}_b - p_b + t_b^p \le t'^p_b$	$\beta_s > \frac{p_s^*}{p_s - p_s^*}$	(No Sale, Buy)
		$\beta_s < \frac{p_s}{p_s - p_s^*}$	(Sale, Not Buy)
$\overline{p}_b < p_b^*$	$\overline{p}_b - p_b + t_b^p > t'^p_b$	$\beta_s > -\frac{p_s}{p_s - p_s^*}$	(No Sale, Not Buy)
		$\beta_s < -\frac{p_s}{p_s - p_s^*}$	(Sale, Buy)
	$\overline{p}_b - p_b + t_b^p \le t'^p_b$	$\beta_s > 0$	(No Sale, Not Buy)
		$\beta_s < 0$	(Sale, Not Buy)

The general equilibrium conclusion is presented in Table 3.

Proof. We use the backward induction method to prove it. During the whole proving procession, we should remember $p_s<p_s^*$ and $p_b<p_b^*$, since this paper focuses on theprice decreasing situation. The proof method is similar to those of Theorem 3 and 4.We first consider the buyer's utility when the good is not on sale. We have:

1)When $\overline{p}_b - p_b^*> 0$, which means $\overline{p}_b>p_b^*$, for the buyer, it holds $u_2(s_{1,2}, s_{2,1})>u_2(s_{1,2}s_{2,2})$, which means if the good is not on sale, the buyer gets a higher utility if choosing to buy. Then the seller should consider when the good is on sale, what strategy the buyer will choose, so that the seller can figure out their own utility.

When the good is on sale, we can assume that when choosing to buy, the buyer got a bigger amount of total utility. If $\beta_b> 0$, then we have:

$$u_2(s_{1,1}, s_{2,1})>u_2(s_{1,1}s_{2,2}) \Leftrightarrow \overline{p}_b - p_b + t_b^p > t'^p_b$$
$$\Leftrightarrow \overline{p}_b - p_b + \left(\beta_b\,(p_b^* - p_b)\right)^{\alpha} > -\lambda\left(\beta_b\,(p_b^* - p_b)\right)^{\alpha'} \quad (7)$$

If $\beta_b< 0$, then we have:

$$u_2(s_{1,1}, s_{2,1})>u_2(s_{1,1}s_{2,2}) \Leftrightarrow \overline{p}_b - p_b + t_b^p > t'^p_b$$
$$\Leftrightarrow \overline{p}_b - p_b - \lambda(\beta_b\,(p_b^* - p_b))^{\alpha'} > -\beta_b\,(p_b^* - p_b))^{a}. \quad (8)$$

So, the buyer will choose to buy when the good is on sale if β_b satisfies $\overline{p}_b - p_b + t_b^p > t'^p_b$,while he will choose not to buy when the good is on sale if β_b does not satisfy $\overline{p}_b - p_b + t_b^p > t'^p_b$.

Now we discuss the situation where β_b satisfies $\overline{p}_b - p_b + t_b^p > t'^p_b$. Since the seller knows the buyer will choose to buy when the good is on sale or not on sale, as we just discussed, he can compare his own utility when the good is on sale or not. Then we have$(s_{1,m}, (s_{2,1},\ s_{2,1}))$. We first assume that when choosing sale, the seller can get a higher utility. Then we have:

$$u_1(s_{1,1}, s_{2,1})>u_1(s_{1,2}s_{2,1}) \Leftrightarrow -\underline{p}_s - \beta_s p_s^* + (1 + \beta_s)p_s > -\underline{p}_s + p_s^* \Leftrightarrow \beta_s < -1.$$

So, we can see that when $\beta_s< -1$, the seller should choose to sale. So, the equilibrium becomes (Sale, Buy). When $\beta_s> -1$, the seller would choose not for sale, the equilibrium will fall in (No Sale, Buy).

We then discuss the situation where β_b does not satisfy $\overline{p}_b - p_b + t_b^p > t'^p_b$. Under this condiction, the buyer will choose not to buy when the good is on sale, but the buyer will still choose to buy when the good is not on sale as $\overline{p}_b - p_b^*> 0$.Thus,$(s_{1,m}, (s_{2,2}, s_{2,1}))$. This may sound odd, but it often happens to people who want to manifest their status by the high price, especially in a developing country like China. Now the seller should consider his total utility between sale and no sale. If the utility of choosing sale is bigger, we have:

$$u_1(s_{1,1}, s_{2,2}) > u_1(s_{1,2} s_{2,1}) \Leftrightarrow -\underline{p}_s + \beta_s(p_s - p_s^*) > -\underline{p}_s + p_s^* \Leftrightarrow \beta_s < \frac{p_s^*}{p_s - p_s^*}.$$

So, we can see that when $\beta_s < \frac{p_s^*}{p_s - p_s^*}$, the seller will choose sale, and then the equilibrium becomes (Sale, Not Buy); and when$\beta_s > \frac{p_s^*}{p_s - p_s^*}$, the seller will choose no sale, theequilibrium becomes (No Sale, Buy).

2)We then discuss the situation where $\overline{p}_b - p_b^* < 0$, which means $\overline{p}_b < p_b^*$, then forthe buyer, if the good is not on sale, the utility will be bigger when he chooses not to buy. That is, $u_2(s_{1,2}, s_{2,1}) < u_2(s_{1,2} s_{2,2})$. Then the seller should consider when the good is on sale, what strategy the buyer will choose to maximize his total utility, too. Formula (7) is applied again and the result can be quote: the buyer will choose to buy when the good is on sale if $\beta_b$satisfies $\overline{p}_b - p_b + t_b^p > t'^p_b$, while the buyer will choose not to buy when the good is on sale if $\beta_b$does not satisfy $\overline{p}_b - p_b + t_b^p > t'^p_b$.

If $\beta_b$satisfies $\overline{p}_b - p_b + t_b^p > t'^p_b$, then we have ($s_{1,m}$, ($s_{2,1}$, $s_{2,2}$)). The seller should compare his own utility between sale and no sale. If sale is better than no sale, then we have:

$$u_1(s_{1,1}, s_{2,1}) > u_1(s_{1,2}, s_{2,2}) \Leftrightarrow -\underline{p}_s - \beta_s p_s^* + (1 + \beta_s) p_s > -\underline{p}_s \Leftrightarrow \beta_s < \frac{p_s}{p_s^* - p_s}.$$

We can see that if $\beta_s < \frac{p_s}{p_s^* - p_s}$, the seller will choose sale, and then the equilibriumwill be (Sale, Buy); when $\beta_s > \frac{p_s}{p_s^* - p_s}$, the seller will choose no sale, and thus theequilibrium becomes (No Sale, Not Buy).

If β_b does not satisfy $\overline{p}_b - p_b + t_b^p > t'^p_b$,then we have ($s_{1,m}$, ($s_{2,2}$, $s_{2,2}$)). The buyer will choose not to buy when the good is on sale. So, if we assume the seller's utility is bigger when he chooses sale, then we have:

$$u_1(s_{1,1}, s_{2,2}) > u_1(s_{1,2}, s_{2,2}) \Leftrightarrow -\underline{p}_s + \beta_s(p_s - p_s^*) > -\underline{p}_s \Leftrightarrow \beta_s < 0.$$

So, we can see that when β_s<0, the seller should choose sale, and thus the equilibrium is (Sale, Not Buy);
when β_s>0,thesellershouldchoosenosale,andthustheequilibriumbecomes(NoSale,NotBuy).

According to Table 3, we can forecast the equilibrium of the game when a price strategy was taken. For example, if a good was often perceived too expensive without discount, i.e., $\overline{p}_b < p_b^*$, and its target customers' β satisfies $\overline{p}_b - p_b + t_b^p > t'^p_b$, and the company's $\beta_s$is less than$\frac{p_s}{p_s^* - p_s}$, then we can forecast the equilibrium is (Sale, Buy),which means that the company will choose sale, and the customer will choose to buy. In the next section, we present an example of this circumstance.

SINGLES' DAY SHOPPING FESTIVAL

With the new game model we developed, we can explain the amazing success of the Tmall.com (the biggest online shopping website in China) on Singles' Day Shopping Festival. On the Singles' Day of 2015 (i.e., 11 November 2015), the total sales were 91,200 million Chinese Yuan, or say 143 billion USA dollars. This sales broke the Guinness World Records [4]. It was reported that during the first hour on that day, the sales surpassed the sum sales of "Black Friday" and "Cyber Monday" in America in 2014. Now suppose there is a silk shirt that a customer has been longed for but too expensive to afford. More specifically, it is priced at 300 RMB but a customer can just pay 240 RMB for it. The shirt's company usually offers no discounts except on Singles' Day Festive of Tmall.com. That is, on Singles' Day, customers just need to pay 250 RMB for it. However, for the company, the total cost of this shirt is 150 RMB and so there are still profits for the company. Suppose the company is famous for its brand management and high quality, so doing price promotion can actually harm its brand reputation. Thus, we can assume β_s = 1. Suppose a customer is a student having little money, so he likes to save money and thus we can assign 1 to $\beta_b$as well. Based on function (2), we assign 0.7 to α and α', and assign 2.25 to λ.

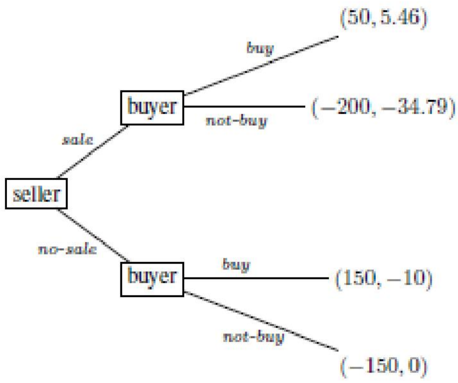

Figure 5: The game tree of trading a shirt in Singles' Day

If we only use acquisition utility to measure the utility of the customer, the customer will not buy the shirt whether the seller chooses sale or not, since the value of the acquisition utility is negative when buying, less than the value of the utility when not buying. However, when considering transaction utility in game model, the outcome is different. The game tree between the company and the customer is shown in Fig. 5.

We use backward induction to find out the equilibrium. That is, the seller chooses$s_{1,m}$, then two choices are left for the buyer in the sub game, i.e., *Buy* or *Not-Buy* in the sub track of the tree in Fig. 5. The buyer will choose the one that brings him a bigger utility. If the company offers discount, the customer will buy the silk shirt because 5.46 > −34.79; otherwise, the customer will not buy the silk shirt because −10 < 0. So we have ($s_{1,m}$, (Buy, Not-buy)). Now the seller can predict that if he chooses $s_{1,1}$= Sale, the outcome of this game is (Sale, Buy), and if he chooses $s_{1,2}$= No-Sale, the outcome of this game is (No-sale, Not-buy). Accordingly, he can compare the outcomes of these two choices and take the better one,s_1^*= Sale, which brings him a bigger utility. Once$s_{1,m}$is settled, player 2's choice is settled, i.e., s_2^*= Buy. Then we have equilibrium (s_1^*,s_2^*) =($s_{1,1}$, $s_{2,1}$).

Therefore, by putting the perceived transaction utility into account of a game, we can see that if a good is discounted appropriately, it will be sold out. Even if the seller earns less in one deal, but it can maximize its total material revenue by selling more.

RELATED WORK

In [22], the authors views a company and residential consumers as two players in a dynamic game, in which the latter's strategy chosen will be the consequence of the former's, and try to find out the optimal pricing strategy. The work in [21] analyses consumer's utilities with different promotion strategies of supermarket retailers. The work in [13] considered the influences of unit marketing expenditure and the unit price charged by the buyer in the end demand of the product in both cooperative and non-cooperative games. In [3], the authors managed to figure out the optimal pricing strategy for a new product in dynamic oligopolies by considering price effect and adoption effect. However, all the above studies concern little about the transaction utility attached to promotion, which makes them less capable to explain impulsive buying behavior in real life. Rather, in this paper, we did put the transaction utility into account, so our model can explain and predict impulsive shopping behavior more accurately.

On the other hand, pricing promotion strategy has been a very active topic in marketing research. In [30], a two-period pricing model is built up for a supply chain to utilize group-buying program to promote its products and derive the equilibrium decisions of the two supply chain members in three different scenarios. Nonetheless, their research neglected the responses of the consumer, thus it is hard to apply the result of the pricing model to markets in real world. In [11][19], a quantities discount pricing model is developed to evaluate the pricing schedule to win the heart of the customer, but none of them considers the psychological influences of the pricing promotion strategy. In [6], the authors presented that different kinds of promotion method influence consumers' choices, but the model cannot be used to estimate the influences to the company's reputation and the consumers' perceived utility. Although in [29] the transaction utility is used to explain why consumers like price promotion so much with a lot of vivid examples, unlike us they did not do any quantities analysis in the framework of game theory.

CONCLUSION

In this paper, we develop a game model with transaction utility, which makes three important theoretical contributions: 1) The rationality in game theory is no longer evaluated simply by material pay-off of the good, but also by the money saved, the reputation of the company that was influenced, and so on. These are important aspects that rational people often consider irrationally in their daily life. So, our model can better explain irrational rationality in human interaction. 2) In game theory, utilities are generic, lacking of clear semantics, so too ambiguous. In our work, utility values are not just some simple numbers without specific semantics, rather they have clear semantics, which can make the outcome more persuasive and practical, reflecting well the situations about price and cost in real trading. 3) The original transaction utility function is applied to the consumers only. However, according to our new definition, this utility function can be applied to not only buyers but also sellers.

Moreover, using our model marketing managers of some online shops can design various sale promotion strategies to attract various customers. Also, after they set a new promotion strategy, they can check it with our model and see whether or not it can stimulate customer to buy. This will bound to help making smart decisions. More specifically, since the parameters in our model are already set in a market and the only variable is the new price after discount, the market manager can use Table 3 to set discount carefully so as to attract more customers to buy.

However, there are still some limitations in our work, which are worthy being removed in future. Firstly, according to framing effect [7], framing price promotion in different ways leads to different consumer perceptions. For example, Chinese subjects are more satisfied with the possibility of refunding than combining offers and percentage effect [16]. Thus, some parameters in our model should be different for not only different types of consumers but also different forms of price promotion. Moreover, among various forms of value function in prospect theory [2], it is worth studying which one is the most suitable for a specific problem discussed in this paper. Finally, since consumer's experience of explosion to promotion activity (including pricing promotion) can change the reference price [10], it requires a facility for automated updating their sale data in time for a discount game we proposed in this paper.

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